CRITICAL POINT TIPS

① To just <u>find</u> the critical points of a function, just solve $\nabla F = (0, 0, \ldots)$. No need to calculate the Hessian & its eigenvalues. However those eigenvalues are needed to determine the <u>type</u> (max min or saddle) of a critical pt. ← plug in the actual point so you have a matrix of numbers!.

② Note that you often have to solve nonlinear equations using a computer (numerically) — eg sagemath or desmos.

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③ When solving systems of equations (eg $\nabla F = (0, 0, \ldots)$) every equation has to be true at the same time.

For example: If one equation gives

$$A = 0 \quad \text{OR} \quad B = 0$$

The next steps should be

(a) Subst. $A=0$ in 2^{nd} (& 3^{rd}) equation(s) & solve that situation.

(b) Subst. $B=0$ in 2^{nd} (& 3^{rd} & ...) equation(s) & solve that situation.

both these yield critical pts (solutions).

Now — I'll show in sagemath how to do these types of computations

We'll do 2.32(b) & find those critical pts — and I'll go further & find a type of one of those points (not asked in homework).

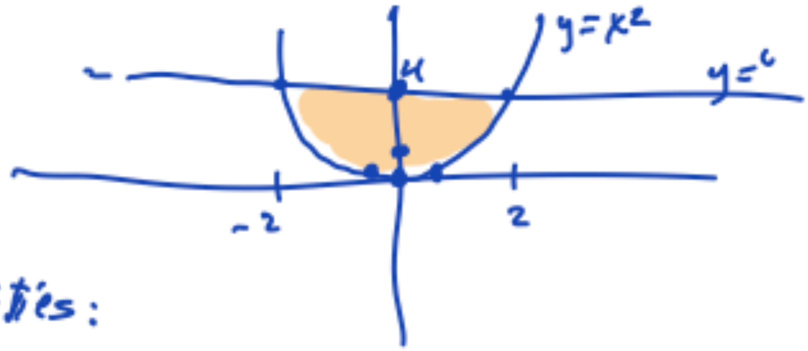
$\boxed{\text{Example of Optimization}}$

Find the abs. maximum & minimum points for the function

$$g(x,y) = 3x^2 + 2y^2 - 4y$$

in the region $R = \{(x,y) : y \le 4 \text{ and } y \ge x^2\}$

Solution:



inside possibilities:

$$\nabla g = (0,0) \qquad (6x, 4y-4) = (0,0)$$

$$x = 0 \qquad 4y - 4 = 0$$

$$y = 1$$

$\boxed{(0,1)}$ actually inside R. ✓

parabola part $\quad y = x^2$

$$g(x,y) = 3x^2 + 2y^2 - 4y$$

$$\underset{y=x^2}{=} 3x^2 + 2x^4 - 4x^2 = 2x^4 - x^2$$

$$f(x) = 2x^4 - x^2, \quad -2 \le x \le 2.$$

$$f'(x) = 8x^3 - 2x = 0$$

$$2x(4x^2 - 1) = 0$$

$$x = 0 \text{ or } x^2 = \frac{1}{4}$$

$$x = \pm\frac{1}{2}$$

$$x = 0, \; x = \frac{1}{2}, \; x = \frac{-1}{2} \qquad (0,0), \left(\tfrac{1}{2}, \tfrac{1}{4}\right), \left(-\tfrac{1}{2}, \tfrac{1}{4}\right).$$

$$y = x^2 \quad y = 0 \quad y = \frac{1}{4}, \; y = 1/4$$

$y = 4$ part:

$$g(x,y) = 3x^2 + 2y^2 - 4y$$

$$h(x) = 3x^2 + 32 - 16 = 3x^2 + 16$$

$$h'(x) = 6x = 0 \Rightarrow x = 0$$
$$y = 4 \qquad (0,4)$$

Last possibilities are corner points

$(2,4)$, $(-2,4)$

To find absolute max & min:

$g(x,y) = 3x^2 + 2y^2 - 4y$

(x,y)	$g(x,y)$
$(0,1)$	-2
$(0,0)$	0
$(\frac{1}{2}, \frac{1}{4})$	$\frac{3}{4} + \frac{2}{16} - 1 =$
$(-\frac{1}{2}, \frac{1}{4})$	
$(0,4)$	
$(-2,4)$	
$(2,4)$	

find greatest & least #s.

.... to be continued....